

METHODOLOGIES FOR FLOODS AND SURGES IN RIVERS¹

D. L. Fread²

SUMMARY

Floods and surges can be mathematically generated by models noted herein which consider the complex natural phenomena involved in the precipitation-runoff process, dam failures, landslide and hurricane generated surges. The mathematical prediction of the changing magnitude and shape of the initially generated flood or surge as it propagates through natural waterways is the function of flood routing models. A synopsis of routing models which are being used in the United States by the engineering profession is presented with emphasis given to the most recent advances associated with those models based on implicit finite difference solutions of the Saint-Venant (flood routing) equations and their application to complex hydraulic processes.

INTRODUCTION

Floods and surges occur in waterways (rivers, reservoirs, or estuaries) as a result of runoff from precipitation (rainfall and/or snow-melt), reservoir releases (spillway flows, hydropower turbine discharges, or dam-failures), landslide generated waves within reservoirs, and tides (astronomical and/or wind generated such as hurricane surges).

The mathematical prediction of the magnitude and temporal properties of the floods and surges has long been of vital concern to man as he has sought to improve the transport of water through man-made or natural waterways and to determine necessary actions to protect life and property from the effects of flooding.

This paper simply notes some of the methodologies for generating the salient characteristics of a flood caused by precipitation runoff, dam failure, landslide or hurricane generated surge. However, the paper is primarily concerned with methodologies for predicting the extent of change in the flood's properties (magnitude, shape, timing) as it propagates through the waterway. Such methodologies are collectively described as flood routing methods. In this paper a general description is given of some recently developed and implemented flood routing methods by the engineering profession as practiced within federal and state

¹ Presented at US-PRC-Japan Trilateral Symposium/Workshop on Engineering for Multiple Natural Hazard Mitigation, Beijing, China, January, 1985.

² Senior Research Hydrologist, National Weather Service, Silver Spring, Maryland, USA

governmental agencies, private consulting firms, and universities throughout the United States. Descriptions are included of recent developments which enable the flood routing methodology to be applicable in complex waterways with hydraulic structures that significantly affect the flood's properties.

FLOOD GENERATION MODELS

Mathematical models for generating the initial flood hydrograph (either discharge or water surface elevation as a function of time) is essential in those cases where the flood originates as a result of complex interaction of several natural phenomena. Such prediction methodologies enable the flood hydrograph to be generated prior to its actual occurrence in the waterway. This provides critical lead time in the case of flood forecasting and warning; or as in the case of engineering flood studies, the effect of the primary causative factors can be determined in terms of the characteristics of the flood occurring at specific points of interest along the waterway.

Precipitation Runoff Floods

The flood hydrograph produced by runoff from precipitation can be generated by mathematical models known as rainfall-runoff models. (See a recent article by Linsley (1982) for an excellent overview of this type of model.) Several models have received considerable use in recent years. Among these are three continuous deterministic conceptual models: 1) the Stanford Watershed Model (Crawford and Linsley, 1966) and its more recent version as described by Johanson, et al. (1980) which use a soil moisture accounting system; 2) the SSARR Model (Rockwood, 1958) which uses a user specified rainfall-runoff relationship and a routing relation to account for watershed, channel, and lake storage; and 3) the National Weather Service River Forecast System (National Weather Service, 1972) which is similar in concept to the Stanford Model but uses a special soil moisture accounting system (Burnash, et al., 1973). Also, among these models are four deterministic conceptual event-type models: 1) the Corps of Engineers HEC-1 Model (Hydrologic Engr. Ctr., 1981) which determines rainfall excess after infiltration loss via several user selected alternative methods including the Soil Conservation Service Curve Number Method (Soil Conservation Service, 1972) while temporal distribution is accomplished by unit hydrograph or kinematic wave techniques; 2) the MITCAT Model (Harley, 1975) which is a distributed parameter model using kinematic wave theory; 3) another distributed kinematic routing rainfall-runoff model by Dawdy, et al. (1978); and 4) the Storm Water Management Model (SWMM) developed for the Environmental Protection Agency (Metcalf and Eddy, Inc. et al., 1971). The continuous models tend to be used on gaged drainage basins while the event-type models are used on both gaged and ungaged basins. Calibration difficulties (uniqueness, efficiency, transferability) are associated with the continuous models while degree of accuracy is a problem with the event-type models.

Dam-Failure Floods

The flood hydrograph produced by the failure of earthen or concrete dams can be predicted by the following empirical techniques: 1) a parametric approach developed by Fread (1977) which considers the time dependent growth of a breach in the dam having constant shape (rectangular, triangular, or trapezoidal) through which the reservoir stored waters are released according to level pool reservoir storage routing through a broad-crested weir breach corrected for downstream tailwater effects; the final width of the breach, the time required to form the breach, the shape of the breach, and the reservoir elevation when breaching commences are empirically determined from historical dam failure information; 2) a simple empirical equation developed by Hagen (1982) which predicts the maximum discharge (Q_m), i.e.,

$$Q_m = 530 (H S_v)^{0.5} \quad (1)$$

in which H is the dam height (ft) and S_v is the reservoir storage volume in acre-ft; the hydrograph is assumed to be triangular with the recession time (hr) given by $T_r = 24.2 S_v / Q_m$; and 3) a simple falling-head weir equation (Fread, 1981), i.e.,

$$Q_m = 3.1 W [C / (T_f + C / \sqrt{H})]^3 \quad (2)$$

in which $C = 23.4 S_a / W$, W is the average breach width (ft) which for earthen dams is defined as $2 < W/H < 4$, and T_f is the time (hr) for breach formation, $0.1 < T_f < 1$, and S_a is the surface area (acre-ft) of the full reservoir; the hydrograph is assumed to be triangular with a time (T_f) for the rising limb and a recession time of $T_r = 24.2 S_v / Q_m - T_f$. The flood hydrograph from breached earthen dams can be predicted by the following two physically based mathematical models: 1) a breach erosion model developed by Ponce and Tsivoglou (1981) which is applicable to overtopping failures of homogeneous earthen dams; the model couples the Meyer-Peter and Muller (M-P-M) sediment transport equation to the one dimensional differential equations of unsteady flow and sediment conservation; the model utilizes a judicious selection of a hydraulic friction factor, a breach width-flow parameter, and a sediment transport coefficient; and 2) a breach erosion model developed by Fread (1984b) which couples the M-P-M sediment transport equation to equations for sediment conservation, reservoir conservation, unsteady uniform flow, and soil slope stability; the model utilizes the dam's soil properties: average particle size, internal friction angle, and cohesive strength for a dam which may have core material which differs from the rest of the dam; failure modes of overtopping, piping and/or pressure collapse can be simulated.

Landslide Generated Surges

The surge resulting from a landslide which rushes into a reservoir, displacing a portion of the reservoir contents and, thereby creating a

very steep water wave which travels up and down the length of the reservoir can be generated by a model described by Fread (1984). The volume of the landslide mass, its porosity, angle of repose, and the time interval over which the landslide occurs are basic input parameters. In the model during small computational time steps, the landslide mass is deposited within the reservoir at its angle of repose and simultaneously the original dimensions of the reservoir are reduced accordingly. The time rate of reduction of the reservoir area creates the wave during the solution of the one-dimensional unsteady flow (Saint-Venant) equations which are defined later herein.

Hurricane Generated Surges

Hurricane-generated surges are predicted by the National Weather Service using the SPLASH Model (Jelesnianski, 1972). SPLASH is a two-dimensional, vertically integrated hydrodynamic model. Externally specified meteorological parameters are utilized to generate the hurricane wind field. These parameters are (a) the radial distance and pressure drop from the storm center to its periphery, and (b) the forward speed of the storm. The wind field submodel empirically computes the maximum wind speed in a stationary storm and generates the wind field by dynamically balancing the computed wind speed, pressure gradient, and inflow angle fields. The computed wind field is then incorporated into the two-dimensional hydrodynamic equation through the wind stress term which drives the model, i.e., causes the development of the storm surge. The governing partial differential hydrodynamic equations of SPLASH are numerically solved using an explicit finite difference technique. Terms relating to the water depth are linearized such that the computed surge height is not added to the undisturbed depth during the computations. Bottom friction is treated using the Ekman principle. The computational grid size is approximately 4 miles. The most recent version has the capability to treat overtopping of finite barrier heights to allow coastal flooding. A similar model (Chen, et al. 1980) is also used by engineering consultants for hurricane surge predictions.

FLOOD ROUTING MODELS

Flood routing may be defined as a mathematical method (model) for predicting the changing magnitude and celerity of a flood wave which propagates through a river, reservoir, or estuary. Commencing with investigations as early as the 17th century, mathematical techniques to predict wave propagation have been continually developed and appear profusely in the engineering literature. The basic theory for the one-dimensional analysis of flood wave propagation was originally developed by Saint-Venant (1871).

Governing Equations

The governing equations for flood routing models are the Saint-Venant equations which consist of a conservation of mass equation:

$$\frac{\partial(AV)}{\partial x} + \frac{\partial A}{\partial t} = 0 \quad (3)$$

and a conservation of momentum equation:

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \left(\frac{\partial h}{\partial x} + S_f \right) = 0 \quad (4)$$

in which t is time, x is distance along the longitudinal axis of the waterway, A is cross-sectional area, V is velocity, g is the gravity acceleration constant, h is the water surface elevation above a datum, and S_f is the friction slope which may be evaluated using a steady flow empirical formula such as the Chezy or Manning equation. Eqs. (3-4) are quasi-linear hyperbolic partial differential equations with two dependent parameters (V and h) and two independent parameters (x and t). A is a known function of h , and S_f is a known function of V and h . No analytical solutions, particularly for practical boundary conditions and cross-section shapes, exist. Derivations of these equations can be found in the literature, e.g. Stoker (1953), Strelkoff (1969), Liggett (1975). A more powerful and useful form of the Saint-Venant equations is their so-called conservation form with additional terms to account for lateral flows, off-channel (dead) storage areas, and wind, i.e.,

$$\frac{\partial Q}{\partial x} + \frac{\partial(A+A_o)}{\partial t} - q = 0 \quad (5)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial(Q^2/A)}{\partial x} + gA \left(\frac{\partial h}{\partial x} + S_f \right) - qv_x + W_f B = 0 \quad (6)$$

where:

$$S_f = \frac{n^2 |Q| Q}{2.2 A^2 R^{4/3}} \quad (7)$$

$$W_f = C_w |V_r| V_r \quad (8)$$

$$V_r = Q/A - V_w \cos \alpha \quad (9)$$

in which Q is discharge, q is lateral inflow (+) or outflow (-), A_o is off-channel (dead) storage cross-sectional area, v_x is the velocity of the lateral inflow in the x -direction of the main channel flow, W_f is the wind factor, n is the Manning friction coefficient, R is the hydraulic radius, C_w is the wind friction coefficient, V_r is the relative velocity between the channel flow and wind, α is the acute angle between the direction of river flow and wind, V_w is the wind velocity which is (-) if aiding the channel flow, and B is the wetted topwidth of the channel.

Simplified Routing Models

Due to the complexities of the Saint-Venant equations their solution was not feasible, and various simplified approximations of flood

wave propagation continued to be developed. Indeed such techniques appear profusely in the engineering literature. An excellent summary of such is presented by Miller and Cunge (1975).

Kinematic Models One popular type of simplified model is the kinematic wave model. The essence of the kinematic model is the use of the following simplified form of the conservation of momentum Eq. (4), i.e.,

$$S_f - S_o = 0 \quad (10)$$

where $\partial h / \partial x = \partial y / \partial x - S_o$ in Eq. (4). Eq. (10) essentially states that the momentum of the unsteady flow is assumed to be the same as that of steady uniform flow as described by the Chezy or Manning equation or some other similar expression in which discharge is a single-valued function of stage, e.g.,

$$A = \alpha Q^\beta \quad (11)$$

in which A is the cross-sectional area, $\alpha = [B / (C^2 S_o)]^{1/3}$, $\beta = 2/3$, C is the Chezy coefficient, and B is the wetted top width of the channel. Combining Eqs. (10-11) and Eq. (3) results in the following nonlinear kinematic wave model (Li, et al., 1975):

$$\frac{\partial Q}{\partial x} + \alpha \beta Q^{\beta-1} \frac{\partial Q}{\partial t} = 0 \quad (12)$$

which can be solved by explicit or implicit finite difference methods, the latter being more efficient in most river applications. The kinematic wave model is limited to applications where single-valued stage-discharge ratings exist, and where backwater effects are insignificant since in kinematic models flow disturbances can only propagate in the downstream direction. Also, the kinematic model modifies the flood wave through attenuation and dispersion via errors inherent in the finite difference solution technique. The phenomenon of numerical damping merely mimics the actual physical damping of a flood wave since there is no mechanism in the basic kinematic equation to cause such damping. The kinematic wave models are very popular in applications to overland flow routing of precipitation runoff.

Diffusion Models Another simplified hydraulic model is the diffusion model which utilizes Eq. (3) and a simplified form of Eq. (4):

$$S_f - \partial h / \partial x = 0 \quad (13)$$

Eq. (13) may be expressed in terms of channel conveyance K_c which is a single-valued function of elevation h, i.e.,

$$Q = K_c (h_x)^{1/2} h_x / |h_x| \quad (14)$$

where $h_x = \partial h / \partial x$. Eq. (14) allows for upstream directed flows. Either explicit (Harrison and Buelter, 1973) or implicit (Brakensiek, 1965)

techniques can be used to solve Eqs. (3 and 14), the latter being much more efficient computationally since it is not restricted to extremely small time steps due to numerical stability constraints of the explicit methods. The nonlinear diffusion wave model is a significant improvement over the kinematic model because of the inclusion in Eq. (13) of the water surface slope term ($\partial h / \partial x$) of Eq. (4). This term allows the diffusion model to describe the attenuation (diffusion effect) of the flood wave. It also allows the specification of a boundary condition at the downstream extremity of the routing reach to account for backwater effects. It does not use the inertial terms (first two terms) of Eq. (4) and, therefore, is limited to slow to moderately rising flood waves in channels of rather uniform geometry. It should be noted that using Eq. (4) rather than Eqs. (13 or 14) whereby the inertial terms are included results in only a 20% increase in computational effort for implicit finite-difference models.

Muskingum-Cunge Models The Muskingum Model first reported by McCarthy in 1938 is based on Eq. (3) written in the following form:

$$\bar{I} - \bar{O} = \Delta S / \Delta t \quad (15)$$

in which ΔS is the change in storage within the reach during a Δt time increment; the storage (S) is assumed to be related to inflow (I) and/or outflow (O), i.e.,

$$S = K[X\bar{I} + (1-X)\bar{O}] \quad (16)$$

If Eq. (15) is expressed in centered finite difference form, the following is obtained:

$$\frac{I_1 + I_2}{2} - \frac{O_1 + O_2}{2} = \frac{S_2 - S_1}{\Delta t} \quad (17)$$

in which the subscript (1) represents time t and (2) represents time $t + \Delta t$. Substituting Eq. (16) into Eq. (17), the following equation for computing O_2 is obtained:

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1 \quad (18)$$

where:

$$C_0 = - (KX - \Delta t/2) / C_3 \quad (19)$$

$$C_1 = (KX + \Delta t/2) / C_3 \quad (20)$$

$$C_2 = (K - KX - \Delta t/2) / C_3 \quad (21)$$

$$C_3 = K - KX + \Delta t/2 \quad (22)$$

Eqs. (18-22) comprise the Muskingum Model. The parameters K and X are determined from observed inflow-outflow hydrographs using such techniques (Singh and McCann, 1980) as: 1) least squares or its equivalent, the graphical method, 2) method of moments, 3) method of cumulants, and

4) direct optimization method. An important variation of the Muskingum model was reported by Cunge (1969) who developed the Muskingum equation using kinematic wave theory including the assumption of a single-valued stage-discharge relation and a four-point implicit finite difference approximation technique. Eq. (18) remains the same, but the following expressions for K and X are determined:

$$K = \Delta x / c \quad (23)$$

$$X = 1/2 [1 - Q_0 / (B_0 c S_0 \Delta x)] \quad (24)$$

where: $c = dQ/dA \quad (25)$

in which c is the kinematic wave speed corresponding to a reference discharge Q_0 , Δx is the reach length, S_0 is the channel bottom slope, and B_0 is channel width corresponding to Q_0 . With K and X defined by Eqs. (23) and (24) the Muskingum Model was shown by Cunge to increase its inherent accuracy from that of kinematic based models to that associated with diffusion models. Ponce and Yevjevich (1978) expanded this method by using variable parameters c and B for temporally varying Q. An important limitation of the Muskingum-Cunge model is its inability to account for backwater effects due to natural channel constrictions, bridges, dams, tides, and large tributary inflows.

Dynamic Wave Models

If the complete Saint-Venant equations are used, the model is known as a dynamic wave model. With the advent of high-speed computers Stoker (1953) first attempted to use the complete Saint-Venant equations for routing floods on the Ohio River. Since then, much effort has been expended on the development of dynamic wave models, and the literature contains many such models. They can be categorized according to direct and characteristic methods of solution of the Saint-Venant equations. In the method of characteristics, the partial differential equations are first transformed into an equivalent set of four ordinary differential equations which are then approximated with finite differences to obtain solutions. Since characteristic methods have not proven advantageous over the direct methods there has not been much continued interest in them and they will not be mentioned further herein. Dynamic models can be classified further as either explicit or implicit, depending on the type of finite difference scheme that is used. Explicit schemes transform the differential equations into a set of easily solved algebraic equations. However, implicit schemes transform the differential equations into a set of algebraic equations which must be solved simultaneously; the set of simultaneous equations may be either linear or nonlinear, the latter requiring an iterative solution procedure.

Explicit Models Explicit finite difference models advance the solution of the Saint-Venant equations point by point along one time line in the x-t solution domain until all the unknowns associated with that time line have been evaluated. Then, the solution is advanced to

the next time line. In an explicit scheme, the spatial derivatives and non-derivative terms are evaluated on the time line where the values of all variables are known. Only the time derivatives contain unknowns. Thus, in an explicit model, two linear algebraic equations are generated from the two Saint-Venant equations at each net point (node). Since the two equations can be solved directly for the unknowns, the equations are described as "explicit." Explicit models are limited to very small time steps due to numerical stability constraints, e.g.,

$$\Delta t < \Delta x / [V + \sqrt{gA/B} + gn^2|V|\Delta x / (2.2 R^{4/3})] \quad (26)$$

The first two terms in the denominator are associated with the well-known Courant condition for stability of explicit schemes in frictionless flow. The third term accounts for the effects of friction. This inequality, or some slight modification thereof, is representative of all explicit models. An inspection of Eq. (26) indicates that the computational time step is substantially reduced as the hydraulic depth (A/B) is increased. Thus, in large rivers, it is not uncommon for time steps on the order of a few minutes or even seconds to be required for numerical stability even though the flood wave may be very gradual, having a duration in the order of weeks. Such small time steps cause the explicit method to be very inefficient in the use of computer time. Another disadvantage of explicit schemes is the requirement of equal Δx distance steps. Among several explicit dynamic models reported in the literature, three that have received considerable use are: 1) the TVA model (Garrison et al., 1969), Army Corps of Engrs. SOCJM Model (Johnson, 1974), and the FAT Model developed by Balloffet (1969).

Implicit Models Implicit models were developed to overcome the limitations on the time step required for explicit models. Implicit models first appeared in the literature in the early 1960's with the work of Preissmann (1961) and later Vasiliev, et al. (1965), Abbott and Ionescu (1967), Baltzer and Lai (1968), Amein and Fang (1970), Quinn and Wylie (1972), Fread (1973, 1974, 1978), and many others. Implicit finite difference schemes advance the solution of the Saint-Venant equations from one time line to the next simultaneously for all points along the time line (i.e., along the x-axis of the waterway). Thus, in an implicit model, a system of $2N$ algebraic equations is generated from the Saint-Venant equations applied simultaneously to the N cross sections along the x-axis. Depending upon the type of implicit finite difference scheme chosen, the system of algebraic equations so generated may be either linear or nonlinear. Implicit models are computationally more complex than explicit models. Depending on the type of implicit scheme (linear or nonlinear), the number of computations during a time step are several times greater than that of an explicit scheme. This extra computational requirement is much greater than that of an explicit scheme. It is prohibitive if the method of solving the system of simultaneous equations is not efficient by taking advantage of the banded-structure of the coefficient matrix of the system of equations. Efficient solution techniques include the following: (1) a compact penta-diagonal

elimination method described by Fread (1971) which makes use of the banded structure of the coefficient matrix of the system of equations, or (2) the double sweep method developed in Europe (Liggett and Cunge, 1975). If the implicit scheme is linear, only one solution of the system of equations is required at each time step. However, if the implicit scheme is nonlinear, an iterative solution is necessary, and this requires one or more solutions of the system of equations at each time step. The use of the Newton-Raphson iterative method for nonlinear systems of equations (Amein and Fang, 1970) provides a very efficient solution if selected convergence criteria are practical. If the Newton-Raphson method is applied only once, the nonlinear implicit model is essentially equivalent to the linearized implicit models with respect to computational effort and performance.

Implicit schemes have generally been four-point, i.e., the conservation of mass and momentum have been applied to the flow existing between two adjacent cross sections. The weighted four-point scheme allows a convenient flexibility in the placement of x-derivative and non-derivative terms between two adjacent time lines in the x-t solution domain. The weighting factor must be equal to or greater than 0.5 to provide unconditional linear stability with respect to time step size, and the accuracy of the scheme generally decreases as the weighting factor approaches unity, i.e., when the terms are expressed entirely at the forward time line. A few six-point schemes have been proposed, e.g., Abbott and Ionescu (1967) and Vasiliev, et al. (1965), but they have the disadvantage of requiring regular Δx intervals whereas the four-point schemes allow variable Δx spacing. Also, the six-point schemes treat the boundary conditions in a more complicated and less desirable manner than the four-point schemes. In implicit 4-pt. schemes, solutions of h and V in Eqs. (3 and 4) or h and Q in Eqs. (5 and 6) are sought in the discretised x-t solution domain which is represented by a rectangular net of discrete points. The net points (nodes) may be at equal or unequal intervals of Δt and Δx along the t and x axes, respectively. Each node is identified by a subscript (i) which designates the x position and a superscript (j) for the time line. A 4-pt. weighted, implicit difference approximation is used to transform the nonlinear partial differential equations of Saint-Venant into nonlinear algebraic equations. The 4-pt. weighted difference approximations are:

$$\frac{\partial K}{\partial t} = (K_i^{j+1} + K_{i+1}^{j+1} - K_i^j - K_{i+1}^j) / (2\Delta t^j) \quad (27)$$

$$\frac{\partial K}{\partial x} = \theta / \Delta x_i (K_{i+1}^{j+1} - K_i^{j+1}) + (1-\theta) / \Delta x_i (K_{i+1}^j - K_i^j) \quad (28)$$

$$K = 0.5 \theta (K_i^{j+1} + K_{i+1}^{j+1}) + 0.5(1-\theta) (K_i^j + K_{i+1}^j) \quad (29)$$

where K is a dummy parameter representing any variable in the Saint-Venant equations, θ is a weighting factor varying from 0.5 to 1, i is a subscript denoting the sequence number of the cross section or Δx reach,

and j is a superscript denoting the sequence number of the time line in the x - t solution domain. A θ value of 0.5 is known as the "box" scheme while $\theta = 1$ is the "fully implicit" scheme. To insure unconditional linear numerical stability and provide good accuracy, θ values nearer to 0.5 are recommended (Fread, 1974). Accuracy decreases as θ departs from 0.5 and approaches 1.0. This effect becomes more pronounced as the time step size increases. Fread (1974) investigated the numerical properties of the weighted 4-pt. implicit scheme applied to the following simplified form of Eqs. (3) and (4):

$$\frac{\partial h}{\partial t} + D_0 \frac{\partial V}{\partial x} = 0 \quad (30)$$

$$\frac{\partial V}{\partial t} + g \frac{\partial h}{\partial x} + kV = 0 \quad (31)$$

in which $k = \frac{2gV_0}{C^2 D_0}$, (32)

h is the water surface elevation, C is the Chezy friction coefficient, and D_0 and V_0 are initial values of hydraulic depth and velocity, respectively. An expression for stability (in the sense of the von Neumann conjecture that linear operators with variable coefficients are stable if all their localized operators in which the coefficients are taken constant are stable) is given by the following expression:

$$|\lambda| = \left[\frac{1 + (2\theta - 2)^2 a + (\theta - 1)b}{1 + 4\theta^2 a + \theta b} \right]^{1/2} \quad (33)$$

in which $a = gD_0 (\Delta t / \Delta x)^2 \tan^2 (\pi \Delta x / L)$; $b = k \Delta t$; and L = wavelength = wave celerity \times duration. If $|\lambda| < 1$, independent of the values of Δx and Δt , the errors due to truncation and round-off will not grow with time, and the difference equations are unconditionally linearly stable. This is the case when $0.5 < \theta < 1$, although only weakly stable (i.e., $|\lambda| = 1$) when $\theta = 0.5$ and k approaches zero. Accuracy of the weighted 4-pt. scheme depends on the selection of Δt (Fread 1984c), i.e.,

$$\Delta t < 0.11 c Z T_p / \sqrt{D_0} \quad (34)$$

where: $Z = \left[\frac{1 - \epsilon^2}{4\theta^2 \epsilon^2 - (2\theta - 2)^2} \right]^{1/2} \quad (35)$

in which T_p is the time of rise of the flood wave in hours, c is the wave celerity in ft/sec, ϵ is the permissible error ratio, $0.90 < \epsilon < 0.99$, and Δt is the time step in hours.

Two of the more frequently used dynamic wave models are the 4-pt. implicit routing models developed by the National Weather Service, DAMBRK (Fread, 1977, 1984a) for dam-break generated floods and DWOPER (Fread, 1978) for any type of flood. A recently developed model by the

U.S. Geological Survey, BRANCH (Schaffranek et al., 1981), is also a 4-pt. implicit model. The computational requirement of the NWS models are 300-400K storage and 0.001 to 0.004 sec per Δx per Δt on an IBM 360/195 computer.

Routing Model Selection

Flood routing has been an important type of engineering analysis and this importance along with its inherent complexity have resulted in the proliferation of routing models. The literature abounds with a wide spectrum of useable and reasonably accurate mathematical models for flood routing when each is used within the bounds of its limitations. The selection of a channel routing model for a particular application is influenced by the relative importance one places on the following factors: (1) model accuracy; (2) the accuracy required in the application; (3) the type and availability of the required data; (4) the available computational facilities; (5) the computational costs; (6) the extent of flood wave information desired; (7) one's familiarity with a given model; (8) the extent of documentation, range of applicability, and availability of a "canned" or packaged routing model; (9) the complexity of the mathematical formulation if the routing model is to be totally developed from "scratch" (coded for computer); and (10) one's capability and time available to develop a particular type of routing model. Taking all factors into consideration and recognizing that each application may change the relative importance of each factor, it is apparent that there is no universally superior routing model. In the absence of significant backwater effects, the kinematic and Muskingum-Cunge routing models offer the advantage of simplicity. The accuracy considerations restrict these models to applications where the depth-discharge relation is essentially single valued. Ponce et al. (1978) and Fread (1984) have developed similar criteria for their acceptable range of application in non-backwater situations. For kinematic-type models including the Muskingum model, the following criterion will restrict routing errors to less than E percent:

$$T_p S_o^{1.6} / (\phi n^{1.2} q_p^{0.2}) > 0.2/E \quad (35)$$

A similar criterion for the diffusion models including the Muskingum-Cunge model is:

$$T_p S_o^{0.7} n^{0.6} / (\phi' q_p^{0.4}) > 0.003/E \quad (36)$$

where: $\phi = (m+1)^2 / (3m+5) \quad (37)$

$$\phi' = (m+3) / (3m+5) \quad (38)$$

in which T_p is the time of rise (hrs) of the inflow hydrograph, S_o is the channel slope (ft/ft), q_p is a unit-width peak discharge (cfs), n is the Manning coefficient, and m is the cross-section shape factor, $0 < m < 2$, where channel topwidth (B) is described by the power func-

tion $B=ky^m$ in which y is flow depth. Inspection of Eqs. (35) and (36) reveals the importance of the parameters, T_p and S_o . Also, it is apparent that the diffusion models are applicable for a wider range of bottom slopes and hydrographs than the kinematic models. In instances of a gently sloping channel and rapidly rising flood wave, when the combination of S_o and T_p becomes small enough that Eq. (36) cannot be satisfied, dynamic wave models are required. The simple Muskingum-Cunge model can be used effectively in many applications where Eq. (36) is satisfied and backwater effects are not significant. However, as the trend continues for increasing computational speed and storage capabilities of computers at decreasing costs, such accessibility to inexpensive computational resources should increase the feasibility of using the dynamic wave models for a wider range of applications. Among the models reviewed, the dynamic wave routing model based on the complete Saint-Venant equations have the capability to correctly simulate the widest spectrum of wave types and waterway characteristics. The dynamic wave models are preferred over all other models when: (1) backwater effect is important due to tides, significant tributary inflows, dams, and/or bridges; and (2) the upstream propagation of waves can occur from large tides and storm surges or very large tributary inflows. The implicit dynamic wave model is the most efficient and versatile although the most complex of the dynamic wave models.

Flood Routing Complexities

The routing of floods in natural waterways entails many complexities that require special treatment. These include internal boundaries, floodplains with meandering rivers and/or levees, mixed subcritical-supercritical flows, networks of river channels, sediment transport effects, and streamflow-aquifer interactions.

Internal Boundaries There may be locations such as a dam, bridge, or waterfall (short rapids) along a waterway where the Saint-Venant equations are not applicable. At these locations, the flow is rapidly varied rather than gradually varied as necessary for the use of the Saint-Venant equations. Empirical water elevation-discharge relations such as weir-flow can be utilized for simulating rapidly varying flow. Unsteady flows are routed along the waterway including points of rapidly varying flow by utilizing internal boundaries. At internal boundaries, cross sections are specified for the upstream and downstream extremities of the section of waterway where rapidly varying flow occurs. Since, as with any other Δx reach, two equations (the Saint-Venant equations) are required, the internal boundary Δx reach requires two equations. The first of the required equations represents the conservation of mass with negligible time-dependent storage, i.e.,

$$Q_1^{j+1} - Q_{1+1}^{j+1} = 0 \quad (39)$$

The second of the required equations can be any appropriate empirical rapidly varied flow relation between discharge (Q_1) and the upstream and

downstream water surface elevations, e.g., the flow through a dam spillway and/or breach, a bridge, a critical flow section, or the overtopping flow of a bridge embankment.

Floodplain with Meandering River Unsteady flow in a natural river which meanders through a wide floodplain is complicated by large differences in geometric and hydraulic characteristics between the river channel and the floodplain, as well as the extreme differences in the hydraulic roughness coefficient. The flow is further complicated by the meandering channel which causes a longer flow path than that for the floodplain and by portions of the floodplain which act as dead storage areas wherein the flow velocity is negligible. Fread (1976, 1984) developed a modified form of the Saint-Venant equations for routing floods in meandering rivers with floodplains such that the flow in the meandering channel and floodplain are identified separately. Thus, the differences in both hydraulic properties and flow-path distance are taken into account in a physically meaningful way, but one that is one-dimensional in concept. This development differs from conventional one-dimensional treatment of unsteady flows in rivers with floodplains, wherein the flow is either averaged across the total cross-sectional area (channel and floodplain) or the floodplain is treated as off-channel storage and the reach lengths of the channel and floodplain are assumed to be identical.

Levee-Separated Floodplain The interaction of floodplain flows with those of the main river but separated from the latter by levees extending parallel to the river channel on either or both sides can be modelled by combining the Saint-Venant equations with broad-crested weir flow equations corrected for submergence effects as described by Fread (1983a). The floodplain may connect back into the main river channel; it may be disconnected as in the case of a floodplain contained within a ringed levee where the floodplain flow is ponded with no exit. The flow may also pass from the river to the floodplain through a time-dependent breach in the levee. Depending on the relative computed elevations in the channel and floodplain, the overtopping levee flow can reverse its direction and flow from the floodplain back into the river. The flow transferred across the levee is considered to be lateral inflow or outflow in the Saint-Venant equations.

When the floodplain is further complicated by its division into a number of separate compartments by levees extending perpendicular to the river into the floodplain, a methodology for routing the flows therein is described by Fread (1984) in which flow transferral is accomplished by broad-crested weir flow corrected for submergence effects and the flow within each compartment is routed by the storage Eq. (15).

Mixed Flow When the flow changes with either time or distance along the routing reach from supercritical to subcritical or, conversely, the flow is described as "mixed". Routine application of either explicit or implicit finite-difference versions of the Saint-Venant

equations to such mixed flows results in highly unstable solutions. Fread (1983) described a stable algorithm for treating such flows with the Saint-Venant equations. At each time step during the solution, subreaches are delineated where supercritical flow exists by computing the Froude number at each cross section and grouping consecutive Δx reaches into subcritical or supercritical subreaches during each time step. Then the Saint-Venant equations are applied and solutions obtained for each subreach, commencing with the most upstream subreach and progressing downstream until each subreach has been solved. Appropriate external boundary equations are used for each subreach. Critical flow is used as the downstream boundary condition at the transition from subcritical to supercritical subreaches. The critical depth and computed discharge are used as upstream boundary conditions for the supercritical reach which does not require a downstream boundary condition. The computed discharge at the downstream end of the supercritical reach is used as the upstream boundary condition for the next subcritical subreach.

Channel Network A network of channels presents complications in achieving computational efficiency when using the 4-pt. implicit solution of the Saint-Venant equations. If equations representing the conservation of mass and momentum at the confluence of two channels are used, a matrix is produced in the simultaneous solution procedure whose elements are not contained within the narrow band along the main-diagonal of the matrix. The column location of the elements within the matrix depends on the sequence numbers of the adjacent cross sections at the confluence. The generation of such "off-diagonal" elements produces a "sparse" matrix containing relatively few non-zero elements. Unless special matrix solution techniques are used for the sparse matrix, the computation time required to solve the matrix by conventional matrix solutions techniques is so great as to make the implicit method infeasible. Two types of algorithms have been used for an efficient computational treatment of channel networks.

The first, called the "relaxation" algorithm (Fread, 1973), is restricted to a dendritic (tree-type) network of channels. In this approach no sparse matrix is generated; the matrix is always banded as it is for a single channel reach. During a time step the Saint-Venant equations are solved first for the main channel, and then they are solved for each tributary. The tributary flow at the confluence with the main channel is treated as lateral flow (q) which is first estimated from previous tributary flows when solving the equations for the main river. The tributary flow depends on its upstream boundary condition, lateral inflows along its reach, and water surface elevation at the confluence (downstream boundary for the tributary) which is obtained during the simulation of the main channel. The interdependence of flows in the main channel and its tributaries requires an iterative solution which usually converges in one or two iterations.

The second type of algorithm (Fread, 1983a; Schaffranek et al., 1981) treats a junction of two channels as an internal boundary condi-

tion consisting of Eq. (15) and two expressions for a steady state form of the momentum Eq. (4). This type of algorithm can be used with any natural system of channels (dendritic systems having any order of tributaries; bifurcating channels such as those associated with islands, deltas, flow bypasses between parallel channels, and tributaries joining bifurcated channels). The network algorithm produces a sparse matrix which is solved by a special matrix technique which treats only non-zero elements. The network algorithm is more versatile than the relaxation algorithm.

Sediment Transport Effects A complex interaction of unsteady flow and sediment transport occurs in rivers with sand beds. The river bottom aggrades (raises) and degrades (lowers) itself during the passage of the flood wave and the hydraulic resistance of the river bottom changes simultaneously as the sand bedforms change their configuration. Aggradation and degradation effects have been modelled by Chen and Simons (1975) and Chang (1984) who coupled the sediment conservation equation to implicit solutions of the Saint-Venant equations. The sediment conservation equation is:

$$(1-\lambda) \frac{\partial A}{\partial t} + \frac{\partial Q_s}{\partial x} - q_s = 0 \quad (40)$$

in which λ is the porosity of the bed material, A is the cross-sectional area of the channel, Q_s is volumetric sediment transport rate computed by an appropriate technique, and q_s is the lateral inflow rate of sediment per unit length. Sediment transport and bed friction interaction have been recently investigated for steady flow by Brownlie (1983).

Streamflow-Aquifer Interaction Interaction of streamflow and the groundwater aquifer for floods occurring in channels situated in arid regions can be of sufficient magnitude to affect the river flow by attenuating the peak flow, reducing the wave peak celerity, and extending the recession limb of the river discharge hydrograph. The river flow lost to the aquifer and at times the added flow from the aquifer have been simulated by coupling flood routing models to either one, two, or three-dimensional groundwater models as reported by Pinder and Sauer (1971), Pogge and Chiang (1977) and Freeze (1972). The coupling occurs through the lateral flow term (q) in the Saint-Venant equations. Both explicit and implicit dynamic wave models were coupled to the unsteady saturated porous media equation and in Freeze's model to the saturated-unsaturated equation.

REFERENCES

- Abbott, M.B. and Ionescu, F. 1967. On the Numerical Computation of Nearly Horizontal Flows. *Journal of Hydraulic Research*, Vol. 5., No. 2, pp. 97-117.
- Amein, M. and Fang, C.S. 1970. Implicit Flood Routing in Natural Channels. *Journal of the Hydraulics Division, ASCE*, Vol. 96, No. HY12, pp. 2481-2500.

- Balloffet, A. 1969. One-Dimensional Analysis of Floods and Tides in Open Channels. *Journal of the Hydraulics Division, ASCE*, Vol. 95, No. HY4, pp. 1429-1451.
- Baltzer, R.A. and Lai, C. 1968. Computer Simulation of Unsteady Flow in Waterways. *Journal of the Hydraulics Division, ASCE*, Vol. 94, No. HY4, pp. 1083-1117.
- Brakensiek, D.L. 1965. An Implicit Flood Routing Method. *Hydraulics Division Specialty Conference, ASCE, Tucson, Arizona.*
- Brownlie, W.R. 1983. Flow Depth in Sand-Bed Channels, *Journal of Hydraulics Division, ASCE*, Vol. 109, No. 7, July, pp. 959-990.
- Burnash, R., Ferrel, R., and McGuire, R., 1973. A Generalized Streamflow Simulation System, Conceptual Modeling for Digital Computers, U.S. National Weather Service, Sacramento, California.
- Chang, H.W. 1984. Modeling of River Channel Changes. *Journal of Hydraulic Division, ASCE*, Vol. 110, No. 2, pp. 157-171.
- Chen, Y.H. and Simons, D.B. 1975. Mathematical Modeling of Alluvial Channels. in: *Modeling 75 Symposium on Modeling Techniques*, 2nd Annual Symposium of Waterways, Harbors, and Coastal Engineering Division, American Society of Civil Engineers, Vol. I, pp. 466-483.
- Chen, M.H., Divoky, D.J., Hwang, L.S. 1980. A Numerical Surge Model with Moving-Boundary Treatment, *Proceedings of the National Symposium on Urban Stormwater Management in Coastal Areas*, pp. 195-204, Virginia Tech, Blacksburg, Virginia.
- Crawford, N.H. and Linsley, R.K. 1966. Digital Simulation in Hydrology: Stanford Watershed Model IV, Tech. Report 39, Dept. of Civil Engr., Stanford University.
- Cunge, J.A. 1969. On the Subject of a Flood Propagation Computation Method (Muskingum Method). *Journal of Hydraulic Research*, Vol. 7, No. 2, pp. 205-230.
- Dawdy, D.R., Schaake, J.C. Jr., and Alley, W.M. 1978. Distributed Routing Rainfall-Runoff Model. U.S. Geological Survey Water Resources Investigations 78-90. 146 pp.
- Fread, D.L. 1971. Discussion of Implicit Flood Routing in Natural Channels by M. Amein and C. S. Fang. *Journal of the Hydraulics Division, ASCE*, Vol. 97, No. HY7, pp. 1156-1159.
- Fread, D.L. 1973. Technique for Implicit Dynamic Routing in Rivers with Tributaries. *Water Resour. Res.*, Vol. 9, No. 4, pp. 918-926.
- Fread, D.L. 1974. Numerical Properties of Implicit Four-Point Finite Difference Equations of Unsteady Flow. NOAA Technical Memo NWS HYDRO-18, U.S. Department of Commerce, National Weather Service.
- Fread, D.L. 1976. Flood Routing in Meandering Rivers with Flood Plains. in: *Rivers '76, Symposium on Inland Waterways for Navigation, Flood Control and Water Diversion*, Vol. I, ASCE, Fort Collins, pp. 16-35.
- Fread, D.L. 1977. The Development and Testing of a Dam-Break Flood Forecasting Model, *Proceedings of Dam-Break Flood Modeling Workshop*, U.S. Water Resources Council, Washington, D.C., pp. 164-197.
- Fread, D.L. 1978. NWS Operational Dynamic Wave Model. in: *Proceedings of Verification of Mathematical and Physical Models*, 26th Annual Hydraulics Division Specialty Conference, American Society of Civil Engineers, College Park, Maryland, pp. 455-464.

- Fread, D.L. 1981. Some Limitations of Dam-Breach Flood Routing Models, Preprint 81-525, ASCE National Meeting, St. Louis, Missouri, 14 p.
- Fread, D.L. 1983a. Computational Extensions to Implicit Routing Models, *Frontiers in Hydr. Engr.*, H.T. Shen, Ed., ASCE, New York, pp. 343-348.
- Fread, D.L. 1983b. Applicability Criteria for Kinematic and Diffusion Routing Models, paper presented at AWRA Conference, San Antonio, Texas, October 1983, 16 p.
- Fread, D.L. 1984a. DAMBRK: The NWS Dam-Break Flood Forecasting Model, Office of Hydrology, National Weather Service, NOAA, Silver Spring, Maryland.
- Fread, D.L. 1984b. A Breach Erosion Model for Earthen Dams, Proceedings of Conference on Delineation of Landslide, Flash Flood, and Debris Flow Hazards in Utah, Utah State University, Logan, Utah.
- Fread, D.L. 1984c. Time Step Selection for the Weighted 4-Pt. Implicit Difference Solution of the Saint Venant Equations, Hydrologic Research Laboratory, National Weather Service, NOAA, Silver Spring, Maryland.
- Freeze, R.A. 1972. Role of Subsurface Flow in Generating Surface Runoff, 1. Base Flow Contributions to Channel Flow. *Water Resources Research*, Vol. 8, No. 3, pp. 609-623.
- Garrison, J.M., Granju, J.P. and Price, J.T. 1969. Unsteady Flow Simulation in Rivers and Reservoirs. *Journal of the Hydraulics Division*, ASCE, Vol. 95, No. HY5, pp. 1559-1576.
- Hagen, V.R. 1982. Re-evaluation of Design Floods and Dam Safety, paper presented at 14th ICOLD Congress, Rio de Janeiro.
- Harley, B.M. 1975. MITCAT Catchment Simulation Model, Description and Users Manual, Version 6, Resource Analysis Corp.
- Harrison, A.S. and Bueltel, C.J. 1973. Computer simulation of Missouri River floods. Part I, U.S. Army Corps of Engineers, Missouri River Division, Omaha, Nebraska.
- Hydrologic Engr. Ctr. 1981. HEC-1 Flood Hydrograph Package. Program Users Manual, U.S. Army Corps of Engineers, California.
- Jelesnianski, C.P. 1972. SPLASH (Special Program to List Amplitudes of Surges and Hurricanes) 1. Landfall Storms, NOAA Tech. Memo NWS TDL-46.
- Johansen, R.C., Imhoff, J.C., and Davis, H.H. Jr. 1980. Users Manual for the Hydrological Simulation Program - Fortran (HSPF), Environmental Protection Agency Report EPA-60/9-80.
- Liggett, J.A. 1975. Basic Equations of Unsteady Flow, Unsteady Flow in Open Channels, Vol. I (Eds. K. Mahmood and V. Yevjevich) Chapter 2, pp. 29-62, Water Resources Publications, Fort Collins, Colorado.
- Liggett, J.A. and Cunge, J.A. 1975. Numerical Methods of Solution of the Unsteady Flow Equations. in: K. Mahmood and V. Yevjevich (Editors), Vol. I, Chapter 4, Water Resources Publications, Fort Collins, Colorado, pp. 89-182.
- Linsley, R.K. 1982. Rainfall-Runoff Models - An Overview, Rainfall-Runoff Relationship. ed. V.P. Singh, pp. 3-12., Water Resources Publications, Littleton, Colorado.
- Metcalf and Eddy, Inc. et al. 1971. Storm Water Management Model, Vols. 1-4, Environmental Protection Agency Report 110224DOC.

- Miller, W.A. and Cunge, J.A. 1975. Simplified Equations of Unsteady Flow. in: K. Mahmood and V. Yevjevich (Editors), Unsteady Flow in Open Channels, Vol. I, Chapter 5, Water Resources Publications, Fort Collins, Colorado, pp. 183-257.
- National Weather Service, 1971. National Weather Service River Forecast System Forecast Procedures, NOAA Technical Memo NWS Hydro-14.
- Pinder, G.F. and Sauer, S.P. 1971. Numerical Simulation of Flood Wave Modification Due to Bank Storage Effects. Water Resources Research, Vol. 7, No. 1, pp. 63-70.
- Pogge, E.C. and Chiang, W.-L. 1977. Further Development of a Stream-Aquifer System Model. Kansas Water Resources Research Institute, University of Kansas.
- Ponce, V.M. and Tsivoglou, A.J. 1981. Modeling Gradual Dam Breaches, Journal of the Hydraulics Division, ASCE, Vol. 107, HY6, pp. 829-838.
- Ponce, V.M. and Yevjevich, V. 1978. Muskingum-Cunge Method with Variable Parameters. Journal of the Hydraulics Division, ASCE, Vol. 104, No. HY12, pp. 1663-1667.
- Ponce, V.M., Li, R.M., and Simons, D.B. 1978. Applicability of Kinematic and Diffusion Models. Journal of the Hydraulics Division, ASCE, Vol. 104, No. HY3, pp. 353-360.
- Quinn, F.H. and Wylie, E.B. 1972. Transient Analysis of Detroit River by Implicit Method. Water Resour. Res., Vol. 8, No. 6, pp. 1461-1469.
- Rockwood, D.M. 1958. Columbia Basin Streamflow Routing by Computer. Transactions, ASCE, Vol. 126, Part 4, pp. 32-56.
- Saint-Venant, B.De. 1871. Theory of Unsteady Water Flow, with Application to River Floods and to Propagation of Tides in River Channels. Comptes rendus, Vol. 73, Acad. Science, Paris, pp. 148-154, 237-240.
- Schaffranek, R.W., Baltzer, R.A., and Goldberg, D.E. 1981. A Model for Simulation of Flow in Singular and Interconnected Channels, Chapt. C3, Book 7, Automated Data Processing and Computations, TWRI Series, U.S. Geological Survey, 100 p.
- Singh, V.P. and McCann, R.C. 1980. Some Notes on Muskingum Method of Flood Routing. Journal of Hydrology, Vol. 48, No. 3, pp. 343-361.
- Soil Conservation Service 1972. Hydrology SCS National Engineering Handbook, Sect. 4, Washington, D.C.
- Stoker, J.J. 1953. Numerical Solution of Flood Prediction and River Regulation Problems; Derivation of Basic Theory and Formulation of Numerical Methods of Attack. Report I, New York University Institute of Mathematical Science, Report No. IMM-200.
- Strelkoff, T. 1969. The One-Dimensional Equations of Open-Channel Flow, Journal of Hydraulics Division, ASCE, Vol. 95, No. HY3, pp. 861-874.
- Vasiliev, O.F., Gladyshev, M.T., Pritvits, N.A., and Sudobicher, V.G. 1965. Methods for the Calculation of Shock Waves in Open Channels. in: Proceedings, 11th Congress IAHR, Leningrad, U.S.S.R., paper 3.44.